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THE LAW OF EXCLUDED MIDDLE AND MANY-VALUED LOGIC

(In the present paper I have taken up the argument from many-valued logic against the law of excluded middle for consideration. An attempt has also been made to investigate into the argument.

Firstly, I have stated the argument which refutes the law, then I have attempted to assess the strength of the argument. Wherever the argument appears unsound, I have tried to give a reason in support.

In my critique of the argument, an effort has also been made to throw some light on the motivations behind the denial — that is, what led to the controversy, and how far the denial has weight.)

Sec. 1 : INTRODUCTORY

Rules of classical logic have come down to us essentially from Aristotle.[1] The most basic rules of classical logic are the so-called three laws of thought. The first, the *Law of Identity*, asserts that '*A* is *A*' or 'if a proposition[2] is true then it is true.' The second, the *Law of Contradiction*, asserts that '*A* cannot be both *B* and not-*B*' or 'A proposition cannot be both true and false.' The third, the *Law of Excluded Middle*, asserts that '*A* is either *B* or not-*B* or 'A proposition is either true or false.' These three laws of thought have been proclaimed to be the most fundamental presupposition of all correct reasoning. But some philosophers, Brouwer[3] for example, have questioned their very status as laws. But the principle which has met the most opposition is the law of excluded middle. It has been the subject of doubt and denial throughout the history of logical theory, more than any other orthodox principle of logic.

The arguments for rejecting the law of excluded middle have come mainly from three directions : many-valued logic[4] quantum mechanics[5] and intuitionism[6]. The law is also rejected on the grounds of an epistemological definition of truth[7]. It is hard to face upto the rejection of anything so basic. So I have taken up the argument from many valued logic for consideration.

Sec. 2 : FORMULATION OF THE LAW

The law of excluded middle has been traditionally formulated differently as follows :

(1) Aristotle formulated the law as :

"There can be no intermediate between contradictories; any given predicate must be either affirmed or denied of one subject[8]."

(2) J. S. Mill States :

"The principle of Excluded Middle (or that one of two contradictories must be true) means that an assertion must be *either* true *or* false : either the affirmative is true, or otherwise the negative is true, which means that the affirmative is false[9]."

(3) W. V. Quine picturises the law as :

(i) Every closed sentence[10] is true or false, or

(ii) Every closed sentence or its negation is true, or

(iii) Every closed sentence is true or not true."[11]

(4) Nicholas Rescher writes :

' " The " 'Law of Excluded Middle' is generally given....as the principle that everything is either *A* or not-*A*, or in a somewhat different formulation, that one member of the pair "*B* is *A*" and "*B* is not-*A*" must be true, so that both cannot be false."[12]

In fact, Rescher gives *five* versions of the law. They are :

"(i) $\propto \vee \neg \propto$

(ii) A proposition must be either true or else false

$(/p/ = T) - \text{or} - (/p/ = F)$.

(iii) Of a propostion and its contradictory *at least one* must-be true.

$(/p/= T) - \text{or} - (/p/ = T)$

or equivalently

if $/p/ \neq T$, then $/-p/ = T$.

(iv) A proposition cannot be true and its denial fail to be false, or its denial be false and it fail to be true.

A proposition is true if and only if its negation (contradictory or denial) is false, and conversely :

$/p/ = T$ iff $/-p/ = F$

$/-p/ = T$ iff $/p/ = F$.

(v) Every proposition either takes a given truth-value or else does not."[13] The law of excluded middle in its symbolic form is expressed as : $pv \sim p$.

The discussion in the present paper concerns and centres around the general thesis that 'every proposition is either true or false'[14] only, irrespective of the stylistically different formulations of it, for it is this thesis which has been questioned in the history of logical theory.

Sec. 3 : THE LAW OF EXCLUDED MIDDLE AND MANY-VALUED LOGIC

One of the quarters in which the law of excluded middle is contested is many-valued logic. Many-valued logic is like the logic of truth-functions except that it recognizes three or more so-called truth values instead of truth and falsity. This kind of logic was developed somewhat by Peirce[15] and independently later by Lukasiewicz[16].

The simplest many-valued logic is one with three possible truth-values. The three values are called 'truth', 'falsity' and 'something intermediate.' The law of excluded middle is not uniformly designated on these matrices. The following truth-table for '$pv \sim p$' where '1' stands for 'truth', '$\frac{1}{2}$' stands for 'something intermediate' and '0' stands for 'falsity' fails to verify the law :

P	v	$\sim p$
1	1	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
0	1	1

'$pv \sim p$ is no longer valid as it has the value '$\frac{1}{2}$' in its truthtable, when 'p' takes the value '$\frac{1}{2}$'. Hence the law cannot be accepted in a system which has three truth-values, '1', '$\frac{1}{2}$' and '0'.

The law of excluded middle is thus not acceptable in many-valued logic.

On Rescher's view, the general form of the law need not be necessarily abandoned in many-valued logic,[17] but the second version[18] of the five possible alternative versions has to be given up. It cannot be maintained in any of the systems of many-valued logic. Rescher maintains that the deliberate violation of this principle is the very basis of and reason for the construction of systems of many-valued logic. For, this version of the law in effect rules out any other truth-values than truth and falsity. It is the key roadblock to the development of many-valued systems of logic, where more than two truth-values are admitted. Hence, it has to be, inevitably, yielded up; but the others versions of the law may be given up in certain cases. For instance, the first version[19] can be maintained in certain systems of many valued logic; in fact, "$\propto \vee \neg\propto$" is an asserted thesis (tautology) in various such systems[20]. On the other hand, it must also be recongized that this formula may very well fail to secure the status of an asserted thesis (tautology) in other perfectly viable systems of many-valued logic, for example, Lukasiewicz's 3-valued logic.

Sec. 4 : AN APPRAISAL

Before I proceed to examine the strength of the argument, I would like to shed some light on the philosophical motivations which led to the development of many-valued logic.

The denial of the law in many-valued logic is on the grounds that there is a third-value apart from the classical 'true' and 'false'. The introduction of a third-value was motivated by the problem of future-contingents,[21] undecidable or indeterminate, statements, meaningless sentences and denotationless expressions.*

Aristotle first discussed the problem of future-contingents in his *De Interpretatione*[22], for propositions about those events in future that may or may not come to pass are neither true nor false. (For instance, 'There will be a sea-battle tomorrow).' Lukasiewicz[23] introduced his three-valued logic to solve such a problem. He maintained that such propositions do not take the values truth or falsity, but something intermediate. Hallden[24] and Routley[25] adopted a three-valued logic to resolve problems about meaningless sentences. A sentence such as 'Abracadabra is a second intention' is neither true nor false, but meaningless.

* Expression which purport to refer but do not really do so.

Frege[26] maintained that for propositions with subjects that do not have anything actual corresponding to them — such as 'The present king of France is wise' (assuming that France has no king)-—the question of truth and falsity does not arise[27]. So to solve the problem of such denotationless expressions, Smiley[28] suggested that a three-valued logic would be appropriate. To accommodate undecidable or indeterminate statements was another reason for the adoption of a many-valued logic. Kleene[29] assigned a third-value 'undecidable' or 'indeterminate' for statements whose truth and falsity could not be determined.

I propose now to offer a critique of the argument raised against the law.

It is true that a meaningless sentence is neither true nor false, but then, truth and falsity are applicable to meaningful sentences, not to meaningless ones. Only a proposition is said to be true or false, and meaningless sentences do not express propositions.

Further, future contingents and indeterminate or undecidable statements do not show the inapplicability of the law. No doubt there are a large number of intermediate statements between those that we know or even believe to be true and those that we know or even believe to be false, but nevertheless we can still hold that each of those intermediate statements is either true or false. Hence an indeterminate or undecidable statement exhibits only our lack of knowledge about their truth or falsity. It can be very well maintained that an indeterminate or undecidable statement is either true or false, though it is not possible to tell which.

The problem of denotationless expressions can be solved in accordance to Russell's translation[30]. According to him, a sentence such as 'The present king of France is wise' is equivalent to a conjunction of 'There is at least a present king of France' and 'there is atmost one present king of France' and 'he is wise.' If there is no such person then the whole conjunction will be false, as the truth of expression depends on the truth of each one of the conjuncts. Thus the problem poses no threat to the law.

Kleene's three-valued logic[31] is intended to accommodate undecidable mathematical statements, which though either true or false, are neither provable, nor disprovable. For example, Kurt Godel[32] in his paper maintained that there is no complete proof procedure for showing the validity of infinity schemata. On the basis of Kleene's logic, the assignment of the third-value to a wff is not intended to indicate that it is neither true nor false, only that one cannot tell which

Again, often intermediate values are understood as epistemological variants on 'true' and 'false', and not as new truth-values. Prior's interpretation[33] of the values '1', '2', '3' and '4' of a four-valued logic is as follows :

1 = true and purely mathematical (or, true and known to be true).

2 = true but not purely mathematical (or, true but not known to be true).

3 = false but not purely mathematical (or, false but not known to be false)

4 = false and purely mathematical (or, false and known to be false).

On this interpretation, any wff is either true or false, for, those with values '1' or '2' are true, and those with values '3' and '4' are false

Smiley's use of Bochvar's three-valued logic too illustrates the point that a many-valued logic need not require the admission of one or more extra truth-values.[34] For him, assignment of the third value to a wff indicates, not that it has an intermediate truth-value, but that it has no truth-value at all.

So it would not be wrong to agree with Haack when she says :

> ".... it is clear that a many-valued logic needn't require the admission of one or more extra truth-values over and above 'true' and 'false', and, indeed, that it needn't even require the rejection of bivalence." [35] and [36]

Hence, one could use a many-valued system and yet hold both:

(1) There are just two truth-values, 'true' and 'false and

(2) Every wff of the system has just one of these values.

Thus the rejection of the law in many-valued logic falls to the ground.

A through examination of the argument raised against the law thus reveals that the rationale for the rejection of it is quite misguided. Modifications of the true-false dichotomy does not exhibit the failure of the law. The law can be said to hold for any number of truth-values. Of the *n* values, a wff will have only one of the *n* values, and this is what the law implies.

As a matter of fact, one who repudiates 'p v ~ p' is indeed giving up classical negation, or perhaps alternation or both. For when the law of excluded middle fails, the law of double negation also fails. If 'p' is neither true nor false, it is false that 'p' is false; If the law of double negation is held, this would imply that 'p' is true, whereas by hypothesis, 'p' is neither true nor false—consequently, on the basis of this, 'it is false that p is false' is not equivalent to 'p' is true.

"In a realm where no proposition excludes its contradictory, nothing could be asserted as true rather thani ts opposite; assertion and negation would vanish. . . ."[37]

I wish to conclude my paper with the remark that the law of excluded middle is true simply by virtue of our conceptual scheme and the rejection of the law stimulated from the quandaries of many-valued logic is thus not acceptable.

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NOTES

1. Aristotle, *De Interpretatione*, trans. Ackrill, J. L., Oxford University Press, Oxford, 1963.
2. There is no uniform use of the word 'proposition' among logicians and philosophers. Some distinguish between a propostion from a sentence, while others use 'sentence' and 'proposition' interchangeably.
3. Brouwer, L. E. J., 'The untrustworthiness of the principles of logic, '*Tijdochriftvoor wijsbegeerte*, ii, reprinted in *Wiskunde Waarheid, Workelijkheid*, 1908.
4. A system of logic in which each formula has more than two possible truth-values. That is, it is an *n*-valued logic where $2 > n$
5. Quantum mechanics is the branch of mathematical physics that deals with the motion of electrons, protons, neutrons and other sub-atomic particles in atoms and molecules.

6. The doctrine advanced by L. E. J. Brouwer and his followers whose key thesis is that a mathematical entity with a particular property exists only if a constructive existence proof can be given for it.

   (A proof of the existence of a mathematical object having a property *p* which gives an example of such an object or atleast a method by which one could find such an example.)
7. Epstemologically 'truth' can be defined in terms of 'verifiability' alone. A proposition is true if it is empirically verifiable and false if its contradictory is empirically verifiable.
8. Aristotle, *Metaphysics*, trans & ed. Warrington, J., Dent, London, 1956 (p. 149).
9. Mill, J. S., *A System of Logic*, Longman's, London, 1843, (p. 183).
10. A sentence that has no free variable.
11. Quine, W.V., *Philosophy of Logic*, Prentice-Hall, New Jersey, 1970, (p. 83).
12. Rescher, N., *Many-valued Logic*, McGraw Hill, New York, 1969, (p. 148).
13. Ibid, pp. 149-150.
14. The Law of Excluded Middle is also referred to as the principle of bivalence for this very thesis.
15. Peirce, C. S. '*Minute Logic*,' (1902) in Hartshorne, C & Weiss, p., (ed.) *Collected papers*, Harvard University Press, Harvard, 1934.
16. Lukasiewicz, 'On 3-valued Logic' (1920) in McCall, S. (ed.) *Polosh Logic*, Oxford University Press, Oxford, 1967.
17. Rescher, N., Op. Cit., pp. 150-1.
18. See Sec. II.
19. Ibid.
20. Rescher has not mentioned any specific system with regard to this point.
21. A future tense statement about a contingent subject matrer, that is, about a matter in which what will or will not occur is still in some way undecidable.
22. Aristotle, *De Interpretatione*, trans. Ackrill, J. L., Oxford University Press, Oxford, 1963.
23. Lukasiewicz, J., Op. Cit., (p. 53).
24. Hallden, S., *The Logic of Nonsense*, Uppsala Universities Arsskrift, 1949.
25. Routley, R., (1) '*On a Significance Theory*' in Australasian Journal of Philosophy (44), 1966. (2) '*The need for Nonsencse*' in Australasian Journal of Philosophy (47), 1969.

26. Frege, G., 'On Sense and Denoting' (1892) in Geach, P. T. and Black, M., (ed.) *Philosiphical Writings of Gottlob Frege*. Blacwell, Oxford, 1960.

27. The problem of denotationless expressions is solvable within the domain of two valued logic itself.

28. Smiley, T. J., 'Sense without Denotation', Analysis (2) 1960.

29. Kleene, S. C., *Introduction to Metamathematics*, North Holland, Amsterdam, 1962.

30. Russell, B., 'On denoting' in *Mind* (14) 1905.

31. Kleene, S. C., Op. Cit.

32. Godel, K., 'On undecidable propositions of formal mathematical systems' Mimeographed. Princeton, 1934. Printed in Davis (ed.) *The Undecidable*, Hewlett, New York, 1965.

33. Prior, A. N., 'Many-valued and Modal Systems : An intuitive Approach, '*Philosophical Review* (64), 1955.

34. Smiley, T. J., Op. Cit.

35. The law of excluded middle is also referred to as the principle of bivalence.

36. Haack, S., *Philosophy of Logics*, Cambridge University Press, Cambridge (1978), p. 213.

37. Blanshard, B., *Reason and Analysis*, Open Court, Illinois (p. 276).

26. [illegible]
27. The [illegible]
28. [illegible]
29. [illegible]
30. [illegible]
31. [illegible]
32. [illegible]
33. [illegible]
34. [illegible]
35. [illegible]
36. [illegible]
37. [illegible]